On Chaotic Nature of the Emerging European Forex Markets

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This study attempts to analyze the presence deterministic chaos in the forex markets of select European countries namely Bulgaria, Croatia, Czech Republic, Hungary Poland, Romania, Russia, Slovakia and Slovenia. Monthly NEER data ranging from jan-1994 to Dec-2013 is used for the purpose of analysis. A two step methodology is employed where in the first step, non-linear dependence structure in the underlying time series is verified using BDS test. The results show that all the markets under study exhibit non-linear dependence. In the next stage, it is enquired whether this non-linear behavior is due to the presence of chaotic dynamics in the markets. This is achieved by estimating Lyapunov exponents for the time series under analysis. An EGARCH (1, 1) filter is applied to see if the non-linearity could be explained by a GARCH process. From the Lyapunov exponent values, it is found that the GARCH process is unable to explain the forex markets behavior in a satisfying manner. It is concluded that the forex markets under study exhibit deterministic chaotic behavior.

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INTRODUCTION

A complex dynamical system could be defined as the type of system with various interacting parts where the emergent behavior of the system may not be equal to the sum of its component behavior. Chaotic systems are a particular type of complex dynamic systems, which could satisfactorily explain a wide range of phenomenon in many complex systems, including biological and physical .Such systems appears to follow a random behavior, but indeed are part of a deterministic process. Its random nature is given by their characteristic sensitivity to initial conditions that drives the system to unpredictable dynamics. However, in a chaotic system, this non-linear behavior is always limited by a higher deterministic structure. For this reason, there is always an underlying order in the apparent random dynamics.

Chaotic systems are said to be mathematically deterministic because if the initial measurements were certain it would be possible to derive the end point of their trajectories. Contrary to classical mechanics, chaos theory deals with nonlinear feedback forces with multiple cause and effect relationships that can produce unexpected results. In such a situation, a chaotic system cannot be understood by the simple disintegration of the whole into smaller parts.

A second feature that is often cited as typical of chaotic behavior of deterministic systems is the impossibility of predicting the future values of the variable(s) concerned (except possibly for the very near future). This might sound like a contradiction. This is where another important feature of chaos comes in, that is the presence of a strange attractor. An attractor as a closed set A such that every trajectory

starting in A remains in A all the time and such that all trajectories starting sufficiently close to A are attracted to A.

Unlike normal attractors (eg: stable fixed point or a stable periodic motion), a strange attractor has two features. The first is that motions in the set A are non-periodic fluctuations, in contrast to the periodic motions (i.e. regularly cyclical, in which distinct situations are repeated at fixed intervals of time) that can arise (in addition to others) in non-chaotic systems. The second one is that these movements show sensitive dependence on initial conditions. Sensitive dependence on initial conditions (henceforth SDIC) means that even very small differences in the initial conditions give rise to widely different paths after some time interval. In a 'normal' deterministic system, all nearby paths starting very close to one another remain very close in the future. Hence, a sufficiently small measurement error in the initial conditions will not affect our deterministic forecasts. On the contrary, in deterministic systems with SDIC, prediction of the future values of the variable(s) would be possible only if the initial conditions could be measured with infinite precision.

A financial market could be termed as a complex adaptive system, as there are many interacting participants whose decision making evolves over time as they gain experience. The presence of chaos in such a system leads to many implications. A major implication of chaos concerns the use of econometric models in forecasting. No matter to what extent one could reduce the confidence intervals for the estimated model, if the true model is chaotic; the presence of SDIC implies the impossibility of forecasting except for maybe a very short run.

Of the financial markets operating in a country, foreign exchange market is most vital to the country's economy. Fluctuations in the foreign exchange can easily affect various segments of the economy. It could also influence the prevailing monetary policy of the central

bank, which, in turn could have a bearing on price stability and growth.

The behavior of exchange rates traditionally has been examined using the time series and structural approach. The limitations of the two approaches have been well documented in the literature. If a forex market is chaotic in nature, many of the tenets held in esteem by mainstream financial theories such as Efficient Market Hypothesis may not hold true. Hence, it would be of interest to test for the presence of chaos in forex markets. The present study positions itself in this direction and examines the behavior of the forex markets of select central and eastern European countries.

This study attempts to analyze the possible presence of deterministic chaos in foreign exchange markets of the following countries: Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania, Russia, Slovakia and Slovenia. These countries were part of the former Eastern bloc and adopted market reforms around the same time, after the dissolution of USSR. Kumar and Kamaiah (2014) analyzed the possible presence of weak form informational efficiency and found that these markets were not informationally efficient. Presence of non-linear dynamical structure could be a possible reason behind such behaviour. Therefore, the present study could be treated as a continuation to our previous work.

In the analysis part, we first test for non-linear dependence structure using BDS test. In the second part, we test the presence of chaotic behaviour by estimating Lyapunov exponents for the underlying time series. First, a review of literature is presented to find the possible research gap.

LITERATURE REVIEW

Even though the interest in chaos was reawakened in physical sciences in 1960's, it took much time for economists to apply this methodology in their field. Mandelbrot (1963) was the first to offer a constructive

criticism of the existing financial modelling. In his seminal work about the cotton prices, he found out evidences for fat-tailed distributions, volatility clustering and volatility jumps in the return series. It is to be noted that Mandelbrot's work did not analyse the chaotic behaviour of cotton prices per se. His study later developed into the science of fractal geometry, which is closely related with the study of chaotic systems. Apart from Mandelbrot's work, there was not any notable contribution in this area for the next twenty years or so.

The apparent randomness of financial markets led some economists to become interested in chaos theory as a theoretical framework to explain those fluctuations. The first notable work of this kind is the study carried out by Hsieh(1989).He had taken the daily bilateral exchange rate of US dollar against British pound, Canadian dollar, Deutsche mark, Japanese yen and Swiss franc. Hsieh applied BDS test to detect the presence of non linearity in the given series. The presence of non linearity was confirmed. However, it could be explained by a GARCH type process.

Vassilicos (1990) used tick-by-tick Deutsche mark/US dollar data, running from Sunday 9 April 1989 to Saturday 15 April 1989.Ask prices were considered, instead of an average between both ask and bid prices. The idea was to capture even slightest fluctuations. Correlation dimension was calculated for the whole data set, as well as for three subsets. Vassilicos did not find any evidence for low dimensional chaos.

Tata and Vassilicos (1991) used the same data set as in Vassilicos (1990) to calculate the largest Lyapunov exponent for the given series, and found out that the value is not significantly larger than zero. Such a result indicated the absence of chaos in the given data. Their findings gave further evidence for the absence of chaos in high frequency data. They had used the algorithm of Wolf et al (1985) to calculate the Lyapunov exponent.

Brooks (1995) used a sample of over twenty years of daily mid-price spot exchange rates of a set of ten currencies against the British pound. The results of Lyapunov exponent estimation showed no evidence of chaos in the pound exchange rates. In all cases, the largest Lyapunov exponent was found to be negative. Brooks concluded that evidence suggested the presence of some sort of nonlinear determinism in the data, but the possibility of deterministic chaos was ruled out.

Sewell et.al.(1996) examined foreign exchange rates between the US and Korea, Taiwan, Japan, Singapore and Hong Kong for the presence for chaos by employing BDS test and Lyapunov exponent. Their studies failed to establish the presence of Chaos in the markets under analysis.

Sengupta et al (1998) tested for non-linearity in the exchange rates of developing countries from Asia and Latin America, as well as for industrialized countries from Europe and USA. Four types of nonlinear dynamics, e.g. autoregressive heteroscedasticity model, Lorenz-type chaos, hysteresis and the positive feedback model were used in the study. The estimated results provided strong support to the existence of significant nonlinearities in the exchange market dynamics.

Belaire- Franch et al. (2001) analysed the exchange rates of 16 OECD countries from 1957 to 1998 using Recurrence Quantification Analysis (RQA). They had used quarterly exchange rate data for the purpose of the analysis. A filtering procedure with the ARMA model was applied first to remove any kind of linear dependence. RQA was applied on residuals from the ARMA model. The results from the analysis confirmed the presence of chaos in the given data set.

Caporale and Spagnolo's (2004) study showed evidence of nonlinearity in the Indonesian, South Korean, and Thai exchange rates employing the BDS test. They suggested that a pure random walk cannot capture the time series properties of these currencies.

Das and Das (2007) implemented the Lyapunov exponent and surrogate data method to investigate the chaotic behaviour of bilateral foreign exchange rates of twelve countries for the period starting from January 1971 to December 2005. They found an indication of deterministic chaos in all exchange rate series.
Adrangi et al. (2010) conducted a battery of tests (correlation dimension tests, BDS tests, and tests for entropy) for the presence of low-dimension chaos, using the daily bilateral exchange rates of the dollar. The strong evidence of nonlinear dependence in the data was not consistent with chaos. The nonlinear dependencies in the dollar exchange rate returns series were explained by a GARCH type model, and therefore the possibility of a chaotic structure was disproved.
From the literature review, it could be found that there is a serious dearth of studies as far as the developing foreign exchange markets of Europe are concerned. Hence, our study becomes relevant in this context.

DATA AND METHODOLOGY

Log returns of monthly NEER (nominal effective exchange rate) data for all the nine countries ranging from Jan-1994 to Dec-2013 were used for the analytical purpose (data collected from Bank of International Settlements website). NEER is considered instead of bilateral rates as NEER being a weighted average, could possibly hold more information about the markets compared to bilateral rates.
The methodology consists of two parts. In the first part, we test for the possible non-linear dependence structure in the given time series by employing BDS. As the presence of a non-linear structure alone cannot confirm the presence of chaotic structure, we estimate Lyapunov exponent for all the return series as well as for the standardized residuals obtained from an EGARCH (1,1) model. The EGARCH filter is applied to see if the possible non-linear structure

could be explained by a GARCH process. A positive value of Lyapunov exponent confirms the presence of chaotic structure.
The following paragraphs gives description about the methods implemented.

3.1. BDS TEST

W.A. Brock, W. Dechert and J. Scheinkman proposed BDS test in 1987 (Brock, Dechert & Scheinkman, 1987). BDS test is a powerful method for detecting serial dependence in time series. It tests the null hypothesis of independent and identically distributed (I.I.D.) against an unspecified alternative.
The BDS test is based the correlation integral concept. Consider a time series $\{x_t: t=1, 2,\ldots, N\}$,which is a random sample of independent and identically distributed (i.i.d.) observations. The correlation integral $C_m(\varepsilon)$ measures the probability that any two of the points $\{X_i\}$ meet within distance ε from each other in m dimensional phase space, and must equal to the product of the individual probabilities, provided that pairs of points are independent:

$$C_m(\varepsilon) = \prod_{i,j(i\neq j)} p(||X_i - X_j|| < \varepsilon) \; for \; N \to \infty \quad ..(1)$$

if all observations are also identically distributed, then

$$C_m(\varepsilon)=[C_1(\varepsilon)]^m \text{ for } N\to\infty. \quad .(2)$$

The statistic: $B(m,\varepsilon,N) = N^{0.5}[C_m(\varepsilon)-C_1(\varepsilon)]$ (3)
would converge to a normal distribution with zero mean and a variance $V(m,\varepsilon,N)$ which could be consistently estimated from the sample data. The BDS statistic is defined as

$$W(m,\varepsilon,N) = \frac{B(m,\varepsilon,N)}{[V(m,\varepsilon,N)]^{1/2}} \; for \; N \to \infty \quad \ldots(4)$$

The BDS statistic, W, will follow a standard normal distribution. The null hypothesis of BDS test is the testing series is of i.i.d. observations. If the W estimator is larger than the level of significance, we can reject the null hypothesis, that is, the nonlinearity exists in the testing series.

3.2. LYAPUNOV EXPONENT

Analysis of the chaotic behaviour depends on the concept of sensitive dependence to initial conditions (SDIC) and the opinion that chaos will exist if nearby trajectories diverge exponentially. One of the implications the existence of SDIC is the systematic loss of predictability of the system over time. The notion of Lyapunov spectrum is often used to quantify and detect this phenomenon.

Lyapunov exponents are calculated as follows :-

$$\lambda = \lim_{n\to\infty} \ln(||Df^n(x)\vec{v}||)/n \quad (5)$$

where D signifies the derivative, $\|\|$ is the Euclidian norm, f^n is the nth iteration of dynamical system f with initial conditions in point x and $\vec{v}$ is a direction vector. If the largest real part of these exponents is positive then the system exhibits sensitivity to initial conditions. In such a case, a larger magnitude means faster decay in the predictability. This method requires knowledge of the analytical structure of the underlying dynamics. In cases where the true dynamics are not known, the alternative is to devise methods for extracting information about the rates of divergence between nearby orbits from a sequence of observed data. An algorithm suggested by Wolf et al (1985) has been used for this purpose.

The procedure could be explained by defining a line S, as a function of the number of time steps, number of observations, the embedding dimension and radius of the ball B (which is an indicator for the size of the neighborhood):

$$S(\Delta n, N, m, \varepsilon)\frac{1}{N-m+1}\sum_{i_0=1}^{N-m+1} ln\left(\frac{1}{|B(X_{i0})|}\sum_{X_j \epsilon B(X_{i0})} ||x_{(i_0+\Delta n,1)} - x_{(j+\Delta n,1)}||\right) \quad (6)$$

Where, $|B(\cdot)|$ is the total number of neighbours in the neighbourhood B (a ball with diameter ε) of the reference vector X_{i0}. $x_{(i0,1)}$ is the most recent, element in the reference vector, X_{i0} and

$x_{(j0+\Delta n,1)}$ is the first observation outside the time span covered by the reference vector.

The basic idea is to trace the distance in between a reference point X_0 and its neighbour, X_j, after n time steps. Set $d_j(X_0,X_j,n)$ to be this distance in the reconstructed phase space and let $\varepsilon(X_0,X_j)$ denote the initial distance between X_0 and X_j. In this case, $d_j(X_0,X_j,n)$ should grow exponentially by the largest Lyapunov exponent $\lambda_{max}(X_0)$, or as it might be expressed in logarithm scale

$$ln\, d_j(X_0, X_j, n) \approx \lambda_{max}(X_0)n + ln\varepsilon(X_0, X_j) \quad (7)$$

It is proposed that, if this linear pattern is persistent for a number of time steps n, the estimated slope is an estimate for the largest Lyapunov exponent. If the value of Lyapunov exponent is greater than zero, then the underlying time series is considered as chaotic.

4.RESULTS AND DISCUSSION

Results of the BDS test and Lyapunov exponent values are reported in table 1 &2 respectively

Table 1

BDS Test Results

Country Name/M & ε	M=2, E=0.5	M=4,E =1	M=8, E=1.5	M=10,E =2
Bulgaria	9.8899 [0.000]	10.3065 [0.000]	9.7314 [0.000]	8.2600 [0.000]
Croatia	3.6117 [0.000]	2.1230 [0.004]	2.5235 [0.011]	3.729 [0.000]
Czech Republic	6.989 [0.000]	8.8305 [0.000]	8.1587 [0.000]	5.4911 [0.000]
Hungary	6.179 [0.000]	5.1368 [0.000]	5.8136 [0.000]	4.0119 [0.000]
Poland	7.4514 [0.000]	5.8474 [0.000]	5.8267 [0.000]	5.8368 [0.000]

Romania	6.8594 [0.000]	8.4211 [0.000]	8.5409 [0.000]	6.7023 [0.000]
Russia	12.128 [0.000]	12.4576 [0.000]	9.0037 [0.000]	6.6513 [0.000]
Slovakia	5.6727 [0.000]	6.2735 [0.000]	6.6783 [0.000]	4.757 [0.000]
Slovenia	8.247 [0.000]	9.8934 [0.000]	9.2268 [0.000]	6.3747 [0.000]

Source: Authors' calculations
P values are in the [ ]

We estimate the BDS test statistic for all the nine forex markets at the embedding dimensions of 2, 4, 8 and 10. After analyzing the results, we can see that the test statistic reject the null of i.i.d. at all embedding dimensions. Here, it is evident that there is a non-linear dependence structure present in all the forex markets under analysis. However, the presence of non-linearity alone cannot confirm the possibility of chaotic dynamics in a financial market. It could of deterministic type, say GARCH type non-linearity. Hence in the next stage, we estimate Lyapunov exponents for the return series as well as for the standardized residuals from a EGARCH (1,1) model. The objective behind applying an EGARCH filter is to see if the non-linearity could be explained by a GARCH (p,q) process. Further, EGARCH model is used to account for the possibility of asymmetric shocks in the markets under study. The results are shown in table 2.

Table 2

Lyapunov Exponent Values

Country	Lyapunov Exponent [return series]	Lyapunov Exponent [EGARCH(1,1) resid.]
Bulgaria	0.6352	0.5510
Croatia	0.6521	0.5871
Czech Republic	0.5397	0.5642
Hungary	0.7186	0.5597
Poland	0.7202	0.5727
Romania	0.5717	0.5849
Russia	0.7321	0.5894
Slovakia	0.4976	0.5300
Slovenia	0.6589	0.5589

Source: Authors' calculations

From the table, certain facts are visible. The Lyapunov exponent values calculated for all the return series are positive. Here, Russian forex market shows the largest value of Lyapunov exponent, followed by Poland and Hungary. However, we need to see whether the non-linearity could be explained by any other process. Towards this, we apply an EGARCH (1,1) model to all time series, extract the standardized residuals and estimate the Lyapunov exponents for all the standardised residuals. After evaluating the results, we can see that the GARCH process is unable to explain the non-linearity present in the forex markets. The Lyapunov exponents are positive for all the markets even after applying the GARCH filter. Here too, Russian forex market shows the largest amount of divergence, followed by Romania and Croatia. Hence, we could conclude that the forex markets under study are chaotic in nature.

CONCLUSION

This study attempted to analyze the possible presence of chaotic structure in forex markets of Bulgaria, Croatia, Czech Republic, Hungary Poland, Romania, Russia, Slovakia and Slovenia. The analysis was carried out employing the monthly NEER data ranging from jan-1994 to Dec-2013 for all the countries.

A two step methodology was used for this purpose where in the first place the presence of non-linear dependence structure in the markets was tested by employing BDS test. The test results showed that all the markets under study have a non-linear dependence structure present.

In the second stage, presence of deterministic chaotic structure in the forex markets was verified by estimating Lyapunov exponents for both the return series as well as standardized residuals from an EGARCH (1,1) model. The results showed that the EGARCH model is insufficient to explain the non-linear behavior and all the markets under study are chaotic in nature.

Here, the presence for deterministic chaos in the forex markets raises two different issues. First, the models that investors use to make investment decisions. Presence of chaos implies that the performance of the existing models needs to be reevaluated. Here, new modeling methods that consider the chaotic nature of the data might be needed to get better results.

Another concern is related to macroeconomic modeling where exchange rate is usually considered as a variable. There too, one need to consider the chaotic properties of the data while evaluating results obtained from the model. Further research is needed in developing macroeconomic modeling considering the possible chaotic nature of macroeconomic variables.

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